

Essential Elements Math Pacing Guide



December

Background

The Essential Elements Math Pacing Guide was inspired by realizing that there is a small amount of information found on the internet to help support educators who teach those who follow an alternate curriculum for our amazing 1% of the student population in education. I wanted to create something that could help serve as a guide, a support, an understanding of how to hold our students to high academic achievement, just like their regular education peers.

Regular education materials are abundant and come with pacing guides with how to implement the prescribed curriculum that the school decided to buy into. Within those curriculums, a good majority of publishers incorporated how to differentiate Instruction for struggling learners, for English Language Learners and/or English as a Second Language learners. However, there does not seem to be a supplementary curriculum that aligns to how to modify instruction and materials for those who follow the alternate curriculum so the 1% of students with disabilities aligned to the alternate curriculum could also learn a modified version of the same materials as their non-disabled peers in an inclusive setting.

Your partner in education,

Jeanette Nowak

Updated May 2022

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December Outline

Standards covered during December:

- [M.EE.6.EE.1-2](#) - Identify equivalent number sentences.
- [M.EE.7.EE.1](#) - Use the properties of operations as strategies to demonstrate that expressions are equivalent.

According to the Dynamic Learning Maps (DLM) website, these are the commonly tested standards that are used for the DLM assessment.

How to Access Math Instruction and Materials from Unique

1. <https://www.n2y.com/unique-learning-system/>
2. Log in using the provided username and password you received
3. Click on Unique Learning System
4. Click on the three lines →
5. Select Monthly Lessons/Unit Lessons
6. Select Math
 - a. When selecting materials, select PDF icon to save and print



Understanding Differentiated Levels in Unique

- Level 3 Learners – can read text and can participate more independently in the lesson (Independent)
- Level 2 Learners- require pictorial support and require mild to moderate support to participate in the lesson (Supported)
- Level 1 Learners- require extensive supports to participate in the lesson (Participatory).

Measuring Success by the Essential Elements Standards

Students who take DLM assessments are instructed and assessed on *Essential Elements*. Essential Elements are grade-specific expectations about what students with the most significant cognitive disabilities should know and be able to do. The Essential Elements relate to college and career readiness standards for students in the general population.

December Math Pacing Guide 6th Grade

[M.EE.6.EE.1-2](#) - Identify equivalent number sentences.

Learning Goal:

- Level 2-3 – Evaluate if equations are true or false.
- Level 1 – Students combine and compare sets.

Essential Questions:

- Do the two sides of this problem have equal value?
- Is this expression true (equal) or false (not equal)?

Vocabulary:

- **Expression** – Numbers, symbols and operators (such as + and ×) grouped together that show the value of something.
- **Equal** – Exactly the same amount or value.



Mini-Map for M.EE.6.EE.1-2

Subject: Mathematics

Expressions and Equations (EE)

Grade: 6

Learning Outcome

DLM Essential Element	Grade-Level Standard
M.EE.6.EE.1-2 Identify equivalent number sentences.	M.6.EE.1 Write and evaluate numerical expressions involving whole-number exponents. M.6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

Linkage Level Descriptions

Initial Precursor	Distal Precursor	Proximal Precursor	Target	Successor
Combine two or more sets of objects to form a new set. Compare two or more sets containing objects to communicate whether a set has the same, different, or an equal number of objects than the other set.	Demonstrate understanding of addition by combining the objects of two or more sets, and demonstrate understanding of subtraction by removing some objects from a larger set.	Represent addition or subtraction word problems or models with equations (e.g., 8 marbles + 3 marbles = 11 marbles). Recognize that the unknown quantity in an equation is represented using a symbol or letter (e.g., $5 + b = 8$).	Recognize a numerical expression that is equivalent to a given expression (e.g., $3 + 4 + 5$ is equivalent to $4 + 3 + 5$). Evaluate an equation to be true or false by determining whether the numerical value on both sides of an equation is the same or different (e.g., analyze whether $5 + 7 = 8 + 4$).	Recognize equivalent expressions by applying commutative and associative properties of addition (e.g., the expression $5 + 8$ is equal to $8 + 5$ due to the commutative property of addition).

Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?

Understanding how to evaluate equations and recognize equivalent expressions requires a student to be able to recognize that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. The educator presents a set, labels it (e.g., two balls, one marker, three CDs), counts the items, labels it again, and encourages students to use numbers to label and count the separate sets. Then, combine the sets, give it a new label, and count the set.

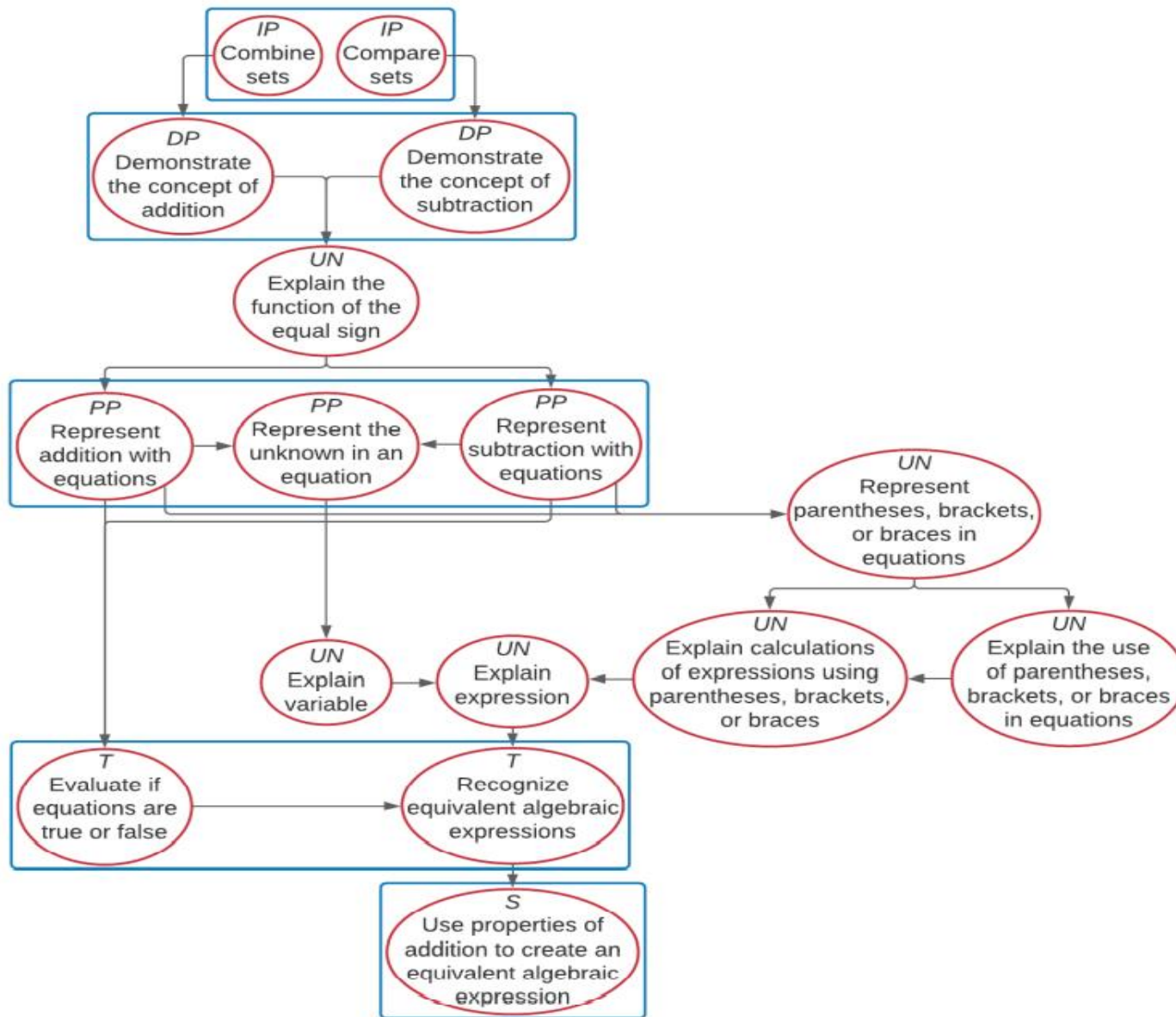
NOTE: Educators can work on the Initial Precursor level using the sets of numbers that students are working with.

How is the Distal Precursor related to the Target?

As students begin to understand labeling and counting small sets, they begin to use the number sequence, and students become more adept at tracking individual objects and can recognize when items are added to a set or when items are taken away. Work on this skill using a variety of sets, labeling and counting the set, and moving items in and out of the set labeling and counting the set again.

NOTE: Educators can work on the Distal Precursor level using the sets of numbers that students working at the Target level are working with.

M.EE.6.EE.1-2 Identify equivalent number sentences.



Map Key	
IP	Initial Precursor
DP	Distal Precursor
PP	Proximal Precursor
T	Target
S	Successor
UN	Untested
Boxes indicate tested nodes	

Rubric of Student Success

[M.EE.6.EE.1-2](#) - Identify equivalent number sentences.

Level 3 Students will...	Level 2 Students will...	Level 1 Students will...
Successor and Target Students will...	Proximal Precursor and Distal Precursor Students will...	Initial Precursor Students will...
Level 3	Level 2	Level 1
<p>Successor</p> <ul style="list-style-type: none"> • Use properties of addition to create an equivalent algebraic expression <p>Target</p> <ul style="list-style-type: none"> • Evaluate if equations are true or false • Recognize equivalent algebraic expressions 	<p>Proximal Precursor</p> <ul style="list-style-type: none"> • Represent addition with equations • Represent the unknown in an equation • Represent subtraction with equations <p>Distal Precursor</p> <ul style="list-style-type: none"> • Demonstrate the concept of addition • Demonstrate the concept of subtraction 	<p>Initial Precursor</p> <ul style="list-style-type: none"> • Combine sets • Compare sets

Instructional Ideas

M.EE.6.EE.1-2 - Identify equivalent number sentences.

Number sentences and equations show a relationship and can be written in different ways.

The big idea is that a number sentence uses numbers and the equal sign to show that two quantities have equal value, whereas a number expression is a math problem that uses numbers and letters to represent variables and an equals sign to show that two quantities have equal value.

- Introduce by asking the essential questions.
- Recognize equivalent algebraic expressions.
- Represent the unknown in the equation.
- Use properties of operation to generate equivalent expressions involving addition, subtraction, multiplication, or division.
- Identify equivalent number sentences.
- Use symbols for equal and not equal.
- Might have to make up your own worksheets but can use the ones provided as inspiration.
- Use manipulatives as needed.
- Students may use a calculator if needed.
- Provide students with their own number line and anchor chart.
- Included worksheets are examples of what to look for when finding additional materials that best fits your student's needs.

Additional Instructional Ideas

- Go to website for additional instructional resources, materials, and activities for lessons:
 - <https://www.msnowakhomeroom.com/3b-equations.html>

Matching Equivalent Addition and Subtraction Number Sentences

I can correctly match equivalent addition and subtraction number sentences. (ACMNA054)



Draw a line to correctly connect the equivalent addition and subtraction number sentences.

9 - 3
10 - 6
16 - 6
9 - 2
7 - 2
7 - 1
20 - 3
15 - 3
8 - 2
7 - 3

3 + 7
2 + 4
3 + 2
12 + 5
3 + 3
5 + 2
2 + 2
3 + 1
4 + 2
1 + 11

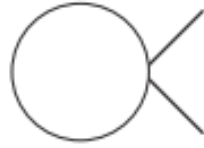
Balancing Equations: Addition to 10

Aim: I can balance equations.

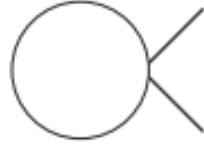
Both sides of an equals sign should make the same total.

Work out the answer to the calculation on the left-hand side and write this in both circles.

Find the missing number on the right, so the calculation makes the number in the circle.



1. $4 + 6 = 7 + \underline{\quad}$



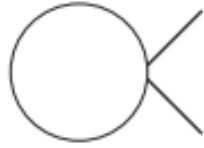
2. $9 + \underline{\quad} = 1 + 9$



3. $1 + 4 = \underline{\quad} + 2$



4. $\underline{\quad} + 7 = 2 + 8$



5. $10 + \underline{\quad} = 3 + 7$



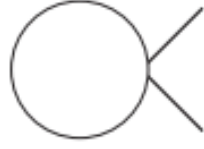
6. $7 + 2 = \underline{\quad} + 9$



7. $10 + \underline{\quad} = 3 + 7$



8. $7 + 2 = \underline{\quad} + 9$

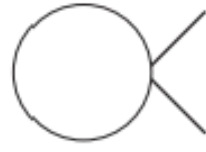


9. $10 + \underline{\quad} = 3 + 7$

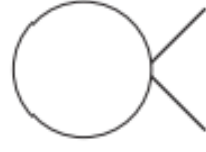
Balancing Equations: Addition to 10



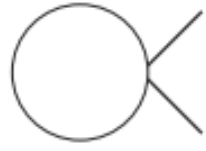
$$7. 4 + 3 = 6 + \underline{\quad}$$



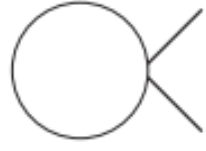
$$8. \underline{\quad} + 5 = 0 + 7$$



$$9. 6 + 2 = 4 + \underline{\quad}$$



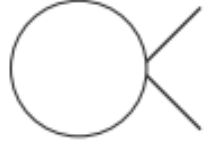
$$10. 9 + 1 = \underline{\quad} + 2$$



$$11. 3 + 6 = 2 + \underline{\quad}$$



$$12. 1 + \underline{\quad} = 2 + 2$$



$$13. 3 + 4 = \underline{\quad} + 0$$



$$14. \underline{\quad} + 8 = 6 + 4$$



$$15. 6 + \underline{\quad} = 5 + 1$$



$$16. 6 + 4 = \underline{\quad} + 7$$



$$17. \underline{\quad} + 5 = 0 + 7$$

Balancing Equations:

Addition to 20

Aim: I can balance equations.

Both sides of an equals sign should make the same total.

Work out the answer to the calculation on the left-hand side and write this in both circles.

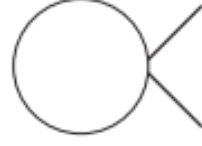
Find the missing number on the right, so the calculation makes the number in the circle.



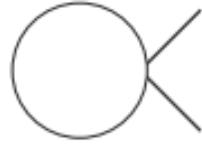
1. $18 + 2 = 19 + \underline{\quad}$



2. $8 + \underline{\quad} = 6 + 6$



3. $17 + 1 = \underline{\quad} + 15$



4. $\underline{\quad} + 6 = 1 + 9$

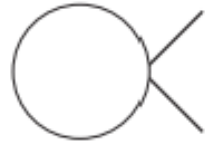


5. $14 + \underline{\quad} = 7 + 10$



6. $7 + 6 = \underline{\quad} + 5$

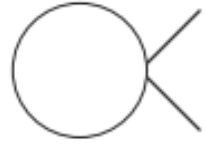
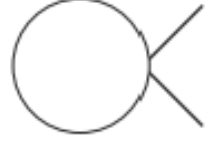


Balancing Equations: Addition to 20

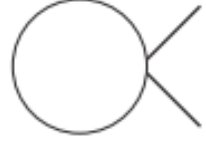
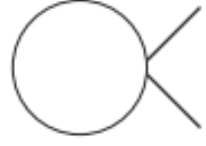
7. $13 + 6 = 17 + \underline{\quad}$



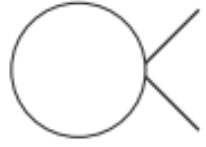
8. $\underline{\quad} + 6 = 2 + 5$



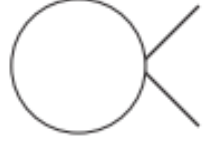
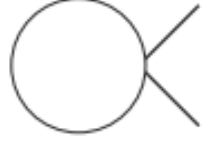
9. $9 + 5 = 6 + \underline{\quad}$



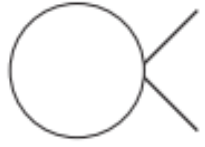
10. $5 + 15 = \underline{\quad} + 8$



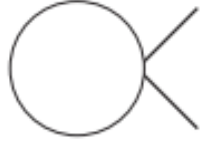
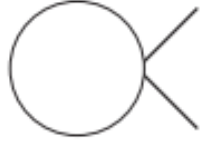
11. $11 + 9 = 10 + \underline{\quad}$



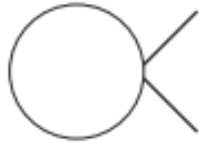
12. $9 + \underline{\quad} = 7 + 11$



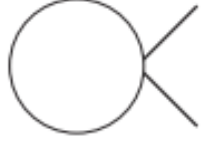
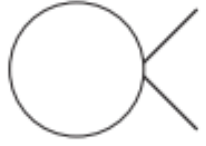
13. $12 + 6 = \underline{\quad} + 17$



14. $\underline{\quad} + 3 = 7 + 9$

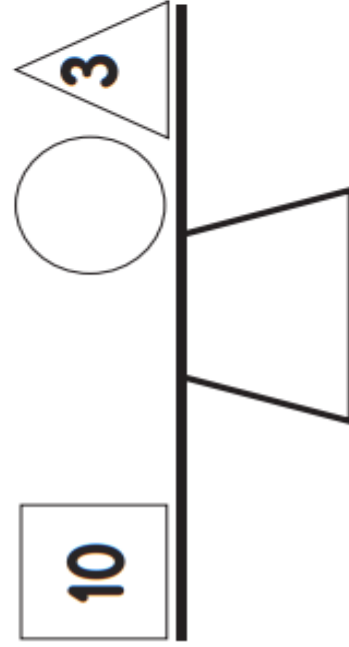
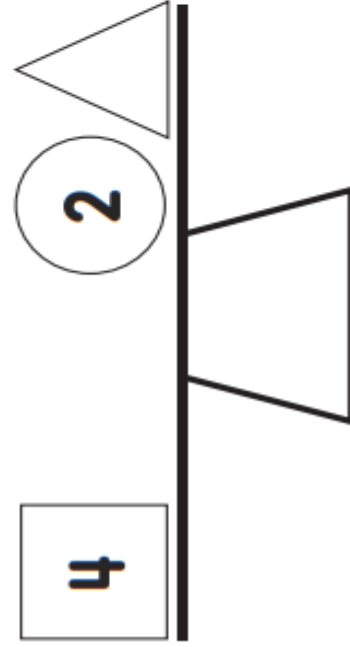
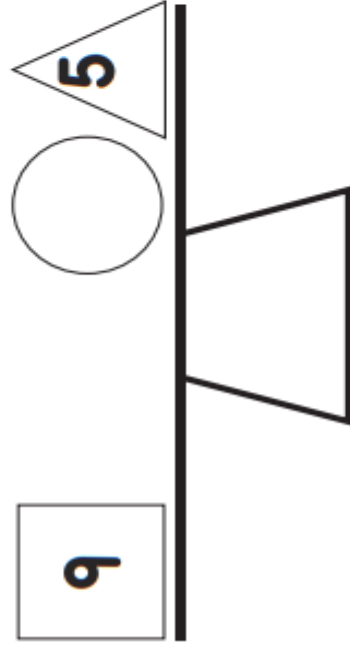
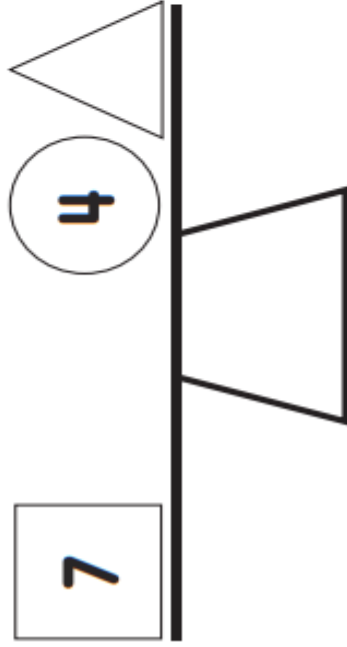


15. $5 + \underline{\quad} = 2 + 9$

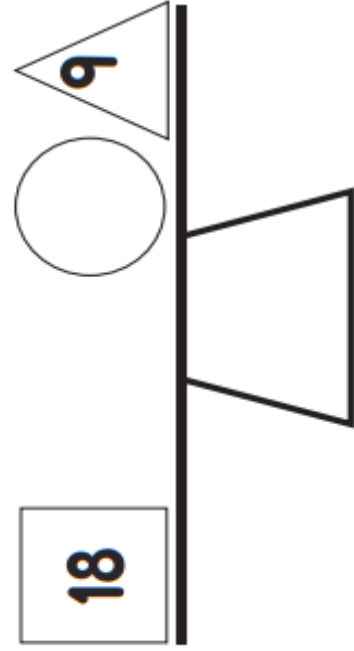
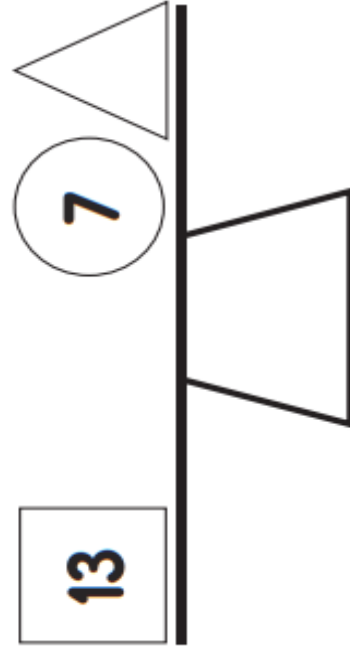
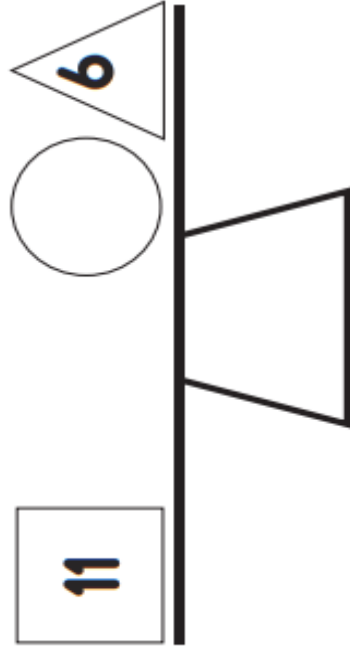
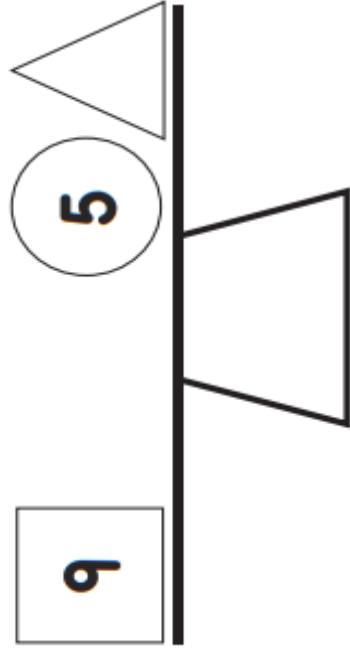


16. $4 + 9 = \underline{\quad} + 12$

Balance the Scales - Up to 10



Balance the Scales - Up to 20



Finding Unknown Quantities in Balanced Number Sentences

1. When a number is added to 10, the answer is the same as 19 minus 6.

10	+	3	=	19	-	6
----	---	---	---	----	---	---

2. When a number is added to 16, the answer is the same as 8 plus 26.

			=			
--	--	--	---	--	--	--

3. When a number is added to 19, the answer is the same as 82 minus 36.

			=			
--	--	--	---	--	--	--

4. When a number is subtracted from 91, the answer is the same as 12 plus 6.

			=			
--	--	--	---	--	--	--

5. When a number is subtracted from 88, the answer is the same as 65 plus 9.

			=			
--	--	--	---	--	--	--

6. When a number is subtracted from 72, the answer is the same as 59 minus 33.

			=			
--	--	--	---	--	--	--

7. When a number is added to 61, the answer is the same as 12 plus 74.

			=			
--	--	--	---	--	--	--



True or False Subtraction Worksheet

Name _____

Look at the equations. Write the answer for each subtraction and find out if the equations are true or false. Check the correct square box for each equation.

$$13 - 3 = 18 - 9$$

☐ true

_____ = _____ ☐ false

$$15 - 6 = 10 - 1$$

☐ true

_____ = _____ ☐ false

$$19 - 4 = 16 - 2$$

☐ true

_____ = _____ ☐ false

$$16 - 8 = 12 - 4$$

☐ true

_____ = _____ ☐ false

$$12 - 7 = 10 - 5$$

☐ true

_____ = _____ ☐ false

$$14 - 3 = 13 - 2$$

☐ true

_____ = _____ ☐ false

$$17 - 8 = 11 - 3$$

☐ true

_____ = _____ ☐ false

$$18 - 5 = 15 - 2$$

☐ true

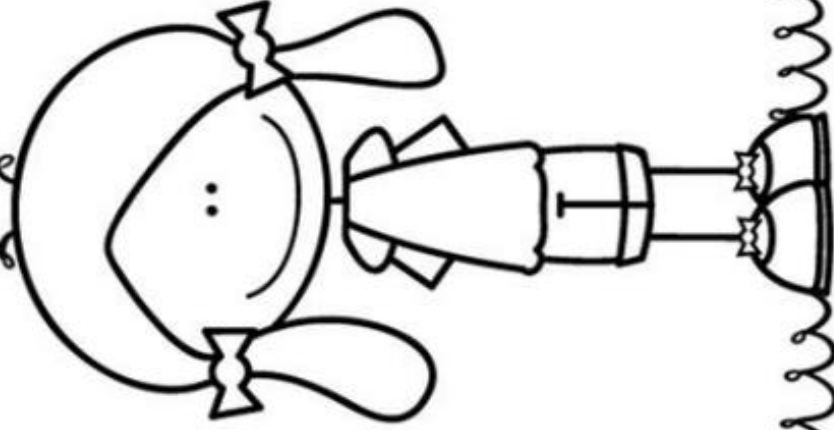
_____ = _____ ☐ false

Name: _____

True/False Subtraction

True

False



© My Teaching Pal



$5-1=3$

$5-3=2$

$4-1=2$

$8-3=4$

$7-6=1$

$6-4=1$

$7-4=4$

$6-3=3$

$4-2=2$

$5-2=3$

Name: _____

True or False Addition

Directions: Read the addition equations below. Are they true? Are both sides of the equal sign the same? Fill in the bubbles that show the true equations.

<p><input checked="" type="radio"/> A $2 + 4 = 4 + 2$</p> <p><input type="radio"/> B $4 + 3 = 4 + 4$</p> <p>Sample</p>	<p><input type="radio"/> A $5 + 3 + 1 = 4 + 5$</p> <p><input type="radio"/> B $4 + 4 + 2 = 8 + 1 + 2$</p> <p>1.</p>
<p><input type="radio"/> A $5 + 5 = 2 + 2 + 3$</p> <p><input type="radio"/> B $3 + 1 = 2 + 2$</p> <p>2.</p>	<p><input type="radio"/> A $2 + 6 = 4 + 5$</p> <p><input type="radio"/> B $6 + 4 + 1 = 7 + 1 + 3$</p> <p>3.</p>
<p><input type="radio"/> A $3 + 3 + 2 = 4 + 4$</p> <p><input type="radio"/> B $3 + 3 + 3 = 4 + 4$</p> <p>4.</p>	<p><input type="radio"/> A $8 + 5 = 10 + 3$</p> <p><input type="radio"/> B $12 + 5 = 10 + 3$</p> <p>5.</p>
<p><input type="radio"/> A $9 + 3 = 3 + 9$</p> <p><input type="radio"/> B $9 + 12 = 8 + 4$</p> <p>6.</p>	<p><input type="radio"/> A $3 + 4 = 5 + 2$</p> <p><input type="radio"/> B $9 + 3 = 11 + 2$</p> <p>7.</p>
<p><input type="radio"/> A $8 + 4 = 2 + 5$</p> <p><input type="radio"/> B $17 + 5 = 5 + 17$</p> <p>8.</p>	<p><input type="radio"/> A $2 + 7 = 8 + 1$</p> <p><input type="radio"/> B $12 + 4 = 2 + 4$</p> <p>9.</p>

December Math Pacing Guide 7th Grade

[M.EE.7.EE.1](#) - Use the properties of operations as strategies to demonstrate that expressions are equivalent.

Learning Goal:

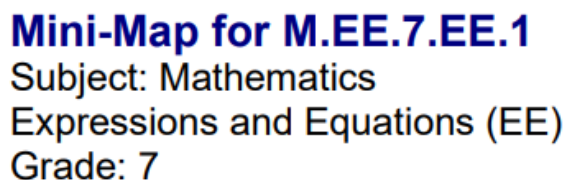
- Level 2-3 – Students will use properties of operations to generate equivalent expressions involving addition and subtraction.
Level 1 – Students will combine and partition sets.

Essential Questions:

- What is the correct order for performing mathematical operations?
- How can the properties of operations be used to determine if two equations are equivalent?
- What is the difference between each of the numbers in this sequence?
- What is the rule for this sequence?

Vocabulary:

- **Commutative property** – you can swap numbers around and still get the same answer when you add or when you multiply.
 $2+4=4+2$ $2 \times 4=4 \times 2$
- **Associative property** – It doesn't matter how we group the numbers (i.e. which we calculate first)



DLM Essential Element	Grade-Level Standard
M.EE.7.EE.1 Use the properties of operations as strategies to demonstrate that expressions are equivalent.	M.7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Initial Precursor	Distal Precursor	Proximal Precursor	Target	Successor
Combine two or more sets of objects or numbers to form a new set. Divide a set of 10 or fewer objects into two or more distinct subsets (e.g., dividing a set containing 10 objects into two subsets containing 4 and 6 objects).	Demonstrate understanding that the sum or product of two numbers remains the same regardless of the order in which numerals are written (e.g., $3 + 4 = 4 + 3$, $2 \times 3 = 3 \times 2$) and that the sum or product of three or more numbers remains the same regardless of the grouping of the numbers [e.g., $(2 + 3) + 5 = 2 + (3 + 5)$, $2 \times (3 \times 5) = (2 \times 3) \times 5$].	Apply commutative (e.g., $3 + 4 = 4 + 3$) and associative [e.g., $(2 + 3) + 5 = 2 + (3 + 5)$] properties of addition to add two or more numbers. Apply commutative (e.g., $3 \times 4 = 4 \times 3$) and associative [e.g., $(10 \times 4) \times 2 = 10 \times (4 \times 2)$] properties of multiplication as strategies to multiply two or more numbers.	Recognize an expression equivalent to a given expression involving addition and subtraction operations by using commutative and associative properties of addition and multiplication {e.g., recognize $[(3 + 4) - (5 \times 6)]$ as an expression equivalent to $[(4 + 3) - (6 \times 5)]$ }. Recognize an expression equivalent to a given expression involving multiplication and division operations by using commutative and associative properties of multiplication and division {e.g., recognize $[(3 \times 4) \div (5 \times 6)]$ as an expression equivalent to $[(4 \times 3) \div (6 \times 5)]$ }. Recognize an expression equivalent to a given expression involving addition and subtraction operations by using commutative and associative properties of addition and multiplication {e.g., recognize $[(3 + 4) - (5 \times 6)]$ as an expression equivalent to $[(4 + 3) - (6 \times 5)]$ }. Recognize an expression equivalent to a given expression involving multiplication and division operations by using commutative and associative properties of multiplication and division {e.g., recognize $[(3 \times 4) \div (5 \times 6)]$ as an expression equivalent to $[(4 \times 3) \div (6 \times 5)]$ }.	Write two equivalent expressions that represent a given real-world problem. For example, "Joe has 5 books, John has 7 books, and Kayla has 8 books. How many books do they have altogether?" Two equivalent expressions that represent this word problem are $(5 + 7) + 8$ and $(7 + 8) + 5$.

Initial Precursor and Distal Precursor Linkage Level Relationships to the Target

How is the Initial Precursor related to the Target?

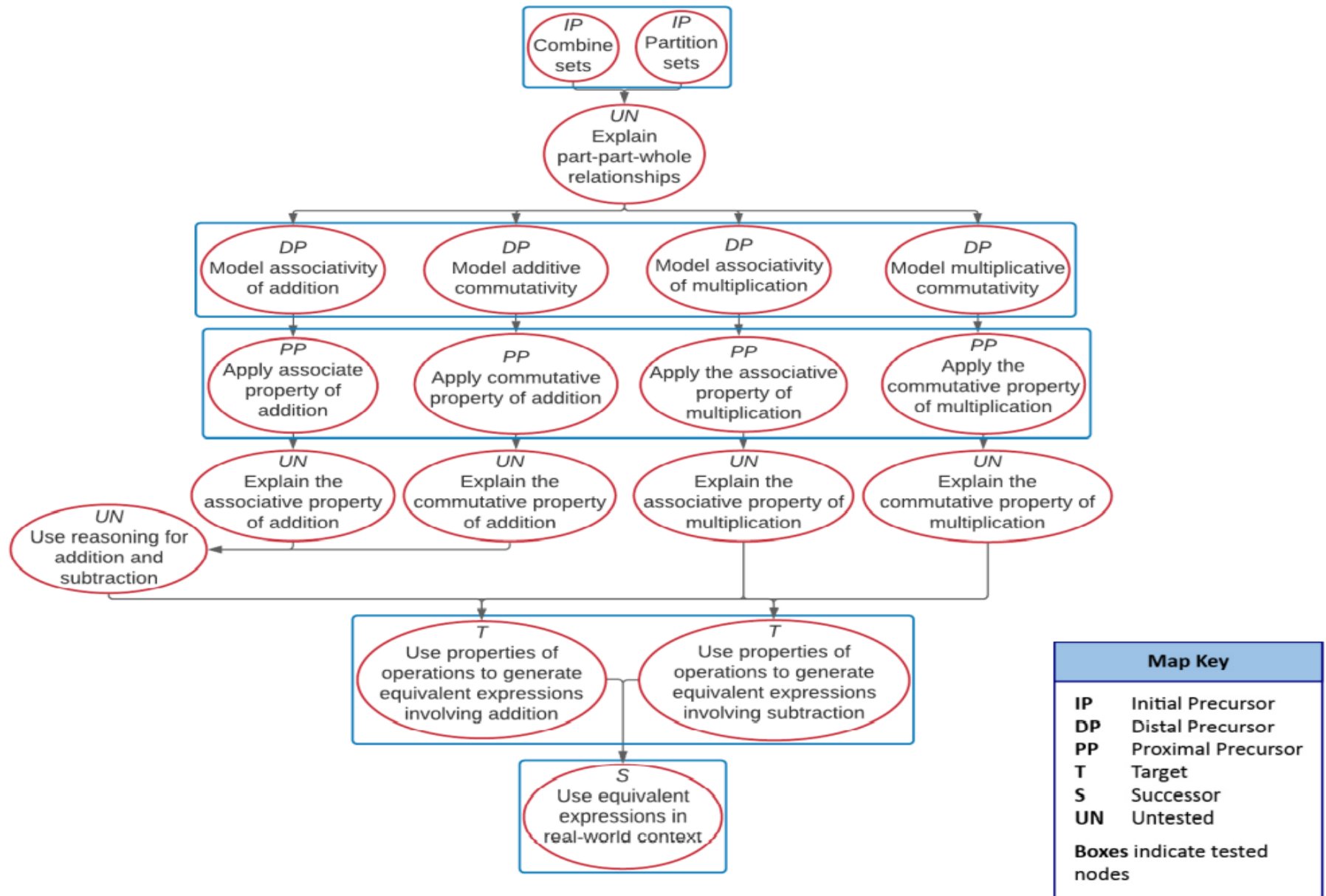
In order to use properties of operations, students begin by counting small units, recognizing that two or more sets or groups of items exist. Work on this skill using a variety of sets. Help students recognize when items are grouped together into a set or separated out. As educators present a set, they label it (e.g., two balls, one marker, three CDs), count the items, label it again, and encourage students to use numerals to label and count the separate sets. The general goal is to explore how the set changes when items are separated out (partitioned) or combined.

How is the Distal Precursor related to the Target?

As students continue developing their understanding of how sets change, educators can use manipulatives to create sets that model the commutative and associative properties of addition and multiplication.

Jeanette Nowak @ msn

M.EE.7.EE.1 Use the properties of operations as strategies to demonstrate that expressions are equivalent.



Rubric of Student Success

[M.EE.7.EE.1](#) - Use the properties of operations as strategies to demonstrate that expressions are equivalent.

Level 3 Students will...	Level 2 Students will...	Level 1 Students will...
Successor and Target Students will...	Proximal Precursor and Distal Precursor Students will...	Initial Precursor Students will...
Level 3	Level 2	Level 1
<p>Successor</p> <ul style="list-style-type: none"> • Use equivalent expressions in real-world context <p>Target</p> <ul style="list-style-type: none"> • Use properties of operations to generate equivalent expressions involving addition • Use properties of operations to generate equivalent expressions involving subtraction 	<p>Proximal Precursor</p> <ul style="list-style-type: none"> • Apply associate property of addition • Apply commutative property of addition • Apply the associative property of multiplication • Apply the commutative property of multiplication <p>Distal Precursor</p> <ul style="list-style-type: none"> • Model associativity of addition • Model additive commutativity • Model associativity of multiplication 	<p>Initial Precursor</p> <ul style="list-style-type: none"> • Combine sets • Partition sets

	<ul style="list-style-type: none">• Model multiplicative commutativity	
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Instructional Ideas

[M.EE.7.EE.1](#) - Use the properties of operations as strategies to demonstrate that expressions are equivalent.

Operations create relationships between numbers.

The big idea is that the commutative and associative properties for addition and multiplication of whole numbers allow computations to be performed flexibly. Subtraction is not commutative or associative for whole numbers. The difference between successive terms in some sequences is constant.

- Introduce by asking the essential questions.
- Apply the properties of operations (i.e., commutative, associative)
- Recognize equivalent expressions (e.g., $A + (B \times C) = (C \times B) + A$, and $(A + B) - C \times (D \times E) = (A + B) - (C \times D) \times E$)
- Identify arithmetic sequence with common difference (e.g., 5, 7, 9, 11, 13, 15 common difference of 2)
- Use manipulatives as needed.
- Students may use a calculator if needed.
- Included worksheets are examples of what to look for when finding additional materials that best fits your student's needs.

Additional Instructional Ideas

- Go to website for additional instructional resources, materials, and activities for lessons:

Commutative Property of Addition Dice

Directions: Roll the dice and draw them in the boxes provided. Write 2 addition sentences using the numbers you rolled. If you roll a 2 and a 3 you would write "2 + 3 = 3 + 2."

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Adding Numbers in Any Order

Aim: I can add numbers to 20 in any order.

It doesn't matter in what order we add numbers together – the answer will always be the same.

$2 + 5$ is the same as $5 + 2$

1. Solve and match up these calculations:

$6 + 2 =$ _____
$7 + 8 =$ _____
$4 + 9 =$ _____
$2 + 9 =$ _____
$1 + 8 =$ _____

$8 + 1 =$ _____
$9 + 4 =$ _____
$8 + 7 =$ _____
$2 + 6 =$ _____
$9 + 2 =$ _____

2. Identify the missing numbers in these matching calculations.

a) $3 + \underline{\quad} = 7$ b) $3 + \underline{\quad} = 12$ c) $4 + \underline{\quad} = 10$

$4 + \underline{\quad} = 7$ $9 + \underline{\quad} = 12$ $6 + \underline{\quad} = 10$

d) $2 + \underline{\quad} = 8$ e) $2 + \underline{\quad} = 13$ f) $1 + \underline{\quad} = 6$

$6 + \underline{\quad} = 8$ $11 + \underline{\quad} = 13$ $5 + \underline{\quad} = 6$

g) $8 + \underline{\quad} = 15$ h) $5 + \underline{\quad} = 9$ i) $7 + \underline{\quad} = 11$

$7 + \underline{\quad} = 15$ $4 + \underline{\quad} = 9$ $4 + \underline{\quad} = 11$

3. Write these calculations in a different way.

a) $7 + 4 =$ _____ b) $5 + 9 =$ _____ c) $9 + 8 =$ _____

The Commutative Law

The Commutative Law says that when you add or multiply numbers, you get the same answer if you swap the numbers round.



$$6 + 3 = 9 = 3 + 6$$



$$4 \times 2 = 8 = 2 \times 4$$

Arithmetic Laws

The Commutative Law

An operation is said to be **commutative** if the order of the operands doesn't affect the result.

Addition is commutative because $a + b \equiv b + a$.

Example

$$3 + 4$$



$$4 + 3$$

In both cases, you have a total of 7 tiles.

Multiplication is commutative because $a \times b \equiv b \times a$.

Example

$$4 \times 5$$



$$5 \times 4$$



In both cases, you have a total of 20 tiles.

Arithmetic Laws

The Associative Law

An operation is said to be **associative** when adding brackets anywhere in a calculation doesn't change the result.

Addition is associative because $a + b + c \equiv (a + b) + c \equiv a + (b + c)$.

Example

$$4 + 5 + 6$$



$$(4 + 5) + 6 = 9 + 6$$



$$4 + (5 + 6) = 4 + 11$$



In all three cases, the result is 15.

Multiplication is associative because $a \times b \times c \equiv (a \times b) \times c \equiv a \times (b \times c)$.

Example

$$(2 \times 3) \times 5$$



$$2 \times (3 \times 5)$$



In both cases, the result is 30.

Multiplying Numbers in Any Order

Aim: I can multiply facts from the 2 times multiplication table in any order.

It doesn't matter in what order we multiply numbers together – the answer will always be the same.

$$2 \times 5 \quad \text{is the same as} \quad 5 \times 2$$

1. Solve and match up these calculations:

$6 \times 2 = \underline{\quad}$
$7 \times 2 = \underline{\quad}$
$4 \times 2 = \underline{\quad}$
$3 \times 2 = \underline{\quad}$
$1 \times 2 = \underline{\quad}$

$2 \times 4 = \underline{\quad}$
$2 \times 6 = \underline{\quad}$
$2 \times 3 = \underline{\quad}$
$2 \times 1 = \underline{\quad}$
$2 \times 7 = \underline{\quad}$

2. Identify the missing numbers in these matching calculations.

a) $3 \times 2 = 6$

b) $5 \times 2 = 10$

c) $4 \times 2 = 8$

$2 \times \underline{\quad} = 6$

$2 \times \underline{\quad} = 10$

$2 \times \underline{\quad} = 8$

d) $1 \times 2 = \underline{\quad}$

a) $2 \times 2 = \underline{\quad}$

f) $7 \times 2 = \underline{\quad}$

$2 \times \underline{\quad} = 2$

$2 \times \underline{\quad} = 4$

$2 \times \underline{\quad} = 14$

g) $9 \times \underline{\quad} = 18$

h) $6 \times \underline{\quad} = 12$

i) $8 \times \underline{\quad} = 16$

$2 \times \underline{\quad} = \underline{\quad}$

$2 \times \underline{\quad} = \underline{\quad}$

$2 \times \underline{\quad} = \underline{\quad}$

3. Write these calculations in a different way.

a) $7 \times 2 = 14$

b) $5 \times 2 = 10$

c) $9 \times 2 = 18$

$\underline{\quad} \times \underline{\quad} = \underline{\quad}$

$\underline{\quad} \times \underline{\quad} = \underline{\quad}$

$\underline{\quad} \times \underline{\quad} = \underline{\quad}$

Commutative Property of Addition

Connect the Matching Problems

$$14 + 82$$

$$8 + 0$$

$$9 + 5$$

$$33 + 97$$

$$10 + 3$$

$$7 + 1$$

$$1 + 7$$

$$5 + 18$$

$$0 + 8$$

$$62 + 45$$

$$45 + 62$$

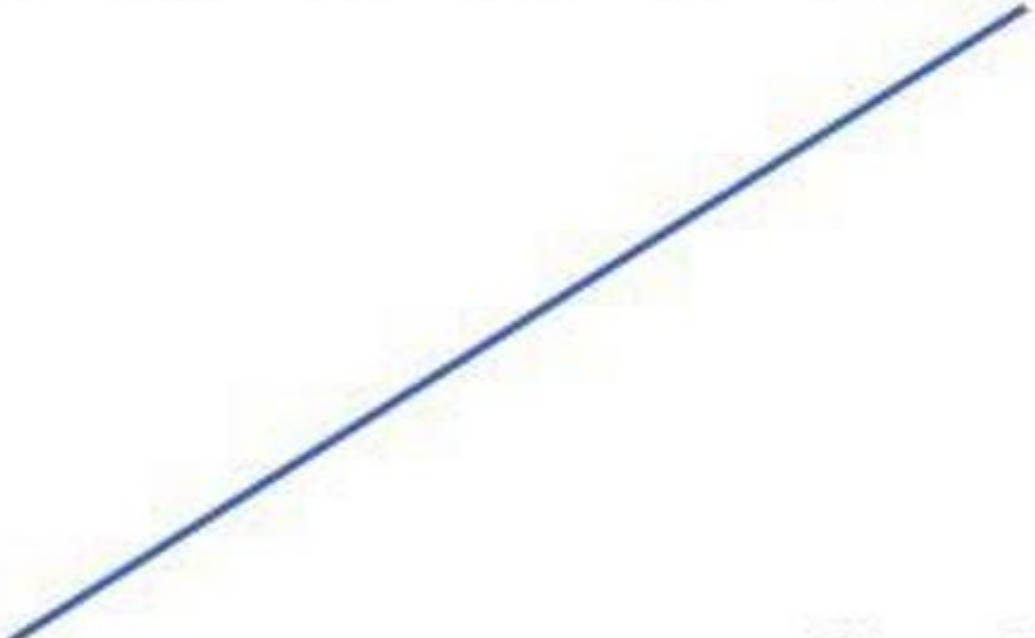
$$3 + 10$$

$$97 + 33$$

$$82 + 14$$

$$18 + 5$$

$$5 + 9$$



Name _____ Date _____

Properties of Multiplication Commutative

One of the multiplication properties is **commutative**, which means that you can multiply numbers in any order and get the same product.

$$A \times B = B \times A$$

Directions: Find the missing number in the equations following the commutative property rule. Then answer the questions below.

$7 \times 5 = 5 \times$

$10 \times 11 = 11 \times$

Julia has four bags of candy. Each bag contains six pieces of candy.
Draw the pieces in each bag.

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How many pieces does Julia have? _____

Tommy has six bags of candies. Each bag contains five pieces of candy.
Draw the pieces in each bag.

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How many pieces does Tommy have? _____

Write the multiplication equations for Julia and Tommy's candy using the commutative property.

	\times		=		\times	
	\times		=		\times	

Multiplication and the Commutative Property

Name: _____

Date: _____

One of the multiplication properties is commutative, which means that you can multiply numbers in any order and get the same product.

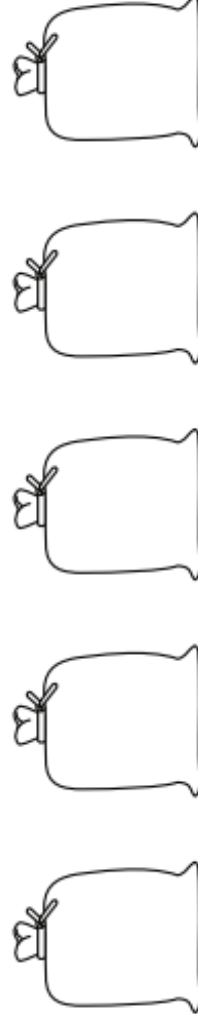
$$A \times B = B \times A$$

Find the missing number in the equations following the commutative property rule. Then answer the questions below.

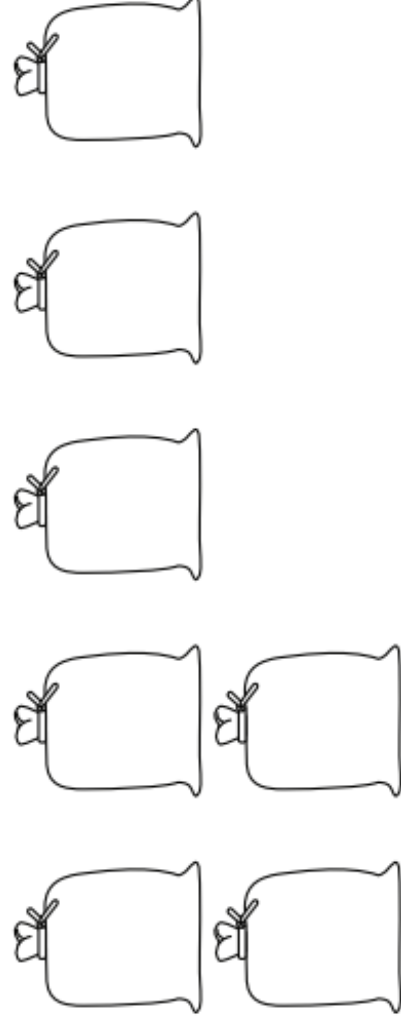
$$1. 5 \times 4 = 4 \times \square$$

$$2. 7 \times 3 = 3 \times \square$$

3. Jenny has five sacks of baby socks. Each bag contains eight socks. Draw the items in each bag. How many socks does Jenny have?



4. Raoul has seven sacks of baby mittens. Each sack contains four mittens. Draw the items in each bag. How many mittens does Raoul have?



5. Write the multiplication equations for each Jenny and Raoul's baby clothes using the commutative property.

$$\begin{array}{l} ___ \times ___ = ___ \times ___ \\ ___ \times ___ = ___ \times ___ \end{array}$$

Name: _____ Date: _____

Multiplication and the Associative Property

One of the multiplication properties is associative, which means you can group the factors in a multiplication equation differently and still get the same product.

$$A \times (B \times C) = (A \times B) \times C$$

Find the missing factor according to the associative property.

1. $5 \times (4 \times 3) = (5 \times 4) \times \boxed{}$

2. $7 \times (3 \times 6) = (7 \times 3) \times \boxed{}$

3. $(30 \times 5) \times 12 = (30 \times 12) \times \boxed{}$

Find the product of these numbers.

4. $6 \times (3 \times 4) = \boxed{} (6 \times 3) \times 4 = \boxed{}$

5. $10 \times (2 \times 5) = 10 \times \boxed{} = \boxed{}$

6. $(10 \times 2) \times 5 = \boxed{} \times 2 = \boxed{}$

Think About It:

7. When you group the factors differently do you get a different product? Explain.

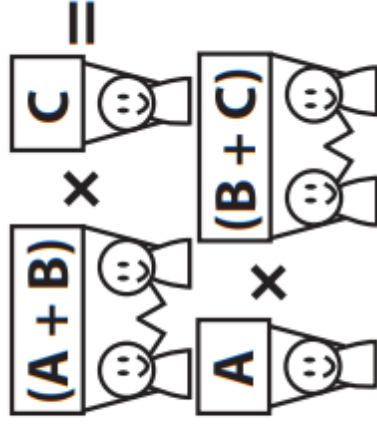
8. How could you change two out of the three factors in an equation and still have the same product?



VOCABULARY CARDS

EL SUPPORT LESSON PLAN: EXPLORE THE ASSOCIATIVE PROPERTY OF MULTIPLICATION

associative



being related or
connected to each other

digit

0, 1, 2, 3, 4,
5, 6, 7, 8, 9

a written symbol for any
of the numbers 0 to 9

parentheses

$(3 \times 4) \times 6$

a pair of marks used
around numbers in math

property



a special characteristic
of something

December Math Pacing Guide 8th Grade

Review 6th and 7th grade standards listed above with 8th graders.

Credits

Websites Used for Worksheets and Lesson Ideas:

- <https://www.education.com>
- <https://www.twinkl.com>
- <https://www.superteacherworksheets.com>
- <https://www.easyteacherworksheets.com>
- <https://www.mathworksheets4kids.com>
- <https://www.math-salamanders.com>
- <https://www.math-drills.com>
- <https://www.mathsisfun.com/definitions/index.html>

Resources Used to Help Create the Pacing Guide:

DLM Essential Elements Unpacking

- <https://www.dlmpd.com/dlm-essential-elements-unpacking>

Instructional Resources for YE Model States

- <https://dynamiclearningmaps.org/instructional-resources-ye/mathematics>

Dynamic Learning Maps

- <https://dynamiclearningmaps.org>

Unique Learning System

- <https://www.n2y.com/unique-learning-system>

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